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Laboratory Permeability Errors from Annular Wall Flow

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ABSTRACT

Laboratory saturated hydraulic conductivity measurement errors introduced by permeameter fluid flow within gaps between sample cores and permeameter walls are estimated through a simple model that idealizes gap flow as occuring within a smooth annulus. The inner annulus boundary fluid velocity is matched to the Darcy velocity within the sample core. The ratio of flow within the annulus to that within the core, Q_{gap}/Q_{core}, is shown to have both cubic and linear dependence on the gap width. Q_{gap}/Q_{core} is also shown to be inversely dependent on both the core permeability and the permeameter radius. Although the model is very idealized, the strong possibility of overestimating permeabilities in laboratory cores is demonstrated. Related concerns include flow in fractured porous media, and potential errors in interpreting solute travel times in laboratory columns and field lysimeters.

Additional Index Words: **permeameter, hydraulic conductivity, preferential flow, fracture flow, solute travel time.**

IN LABORATORY MEASUREMENTS of the saturated hydraulic conductivity, K_s, it is generally assumed that fluid flow occurs exclusively within the sample core. Fluid flow within the annulus bounded by the lateral surface of the core sample and the inner surface of the permeameter tube is generally assumed to be insignificant. In recent years, interest in measuring flow in materials characterized by low hydraulic conductivities has been renewed (e.g., Neuzil, 1986). In measurements of low permeability materials and media subject to bulk volume changes, permeameter fluid flow within gaps between sample walls and permeameter walls can become substantial, leading to erroneously large estimates of K_s. This effect was noted in a work by McNeal and Reeve (1964), in which a method for separating central core flow from wall effects was presented. Significantly larger flux densities due to wall effects were measured in that study. In the present paper, wall flow errors are estimated using the simplifying assumption that the flow occurs within a smooth annular gap. Annular flow errors are expressed in terms of a ratio of gap flow to core flow.

Laminar Flow within an Annulus

Flow between a core sample and the permeameter wall will be described by annular flow along the z direction between two concentric cylinders of length L as shown in Fig. 1. Although it is recognized that this configuration is not likely in actual laboratory permeameters, it is quite likely that gaps spanning varying fractions of the full annular region can occur. The inner cylinder wall of radius R_1 corresponds to the lateral surface of the core. The outer cylinder wall of radius R_2 corresponds to the inner surface of the permeameter tube. The ratio (R_1/R_2) is designated C. The gap width δ is equal to $R_2 - R_1$. The surface at R_1 is idealized as a constant velocity boundary, with the fluid velocity $v_z(R_1)$ matched to the Darcy velocity within the core. The surface at R_2 is treated as a no-slip boundary. End effects at $z = 0$ and $z = L$ are assumed to have negligible influences on the annulus velocity profile. The driving force for flow through the annulus is the hydraulic head gradient, ∇H.

The solution to steady laminar flow in this system is obtainable through a shell momentum balance, in a manner closely following a nearly identical problem described by Bird et al. (1960, p. 51–54). The only difference between these problems is in the treatment of the boundary at R_1, where Bird et al. set $v(R_1) = 0$. In cylindrical coordinates, the differential equation to be solved is

$$\frac{d}{dr}\left[r\mu\frac{dv_z}{dr}\right] = \rho g r\frac{dH}{dz}, \quad [1]$$

where μ is the fluid viscosity, ρ is the fluid density, g is the acceleration of gravity, and H is the hydraulic head. The boundary conditions are

$$v_z(R_1) = -k\frac{\rho g dH}{\mu\, dz}, \quad [2a]$$

$$v_z(R_2) = 0, \quad [2b]$$

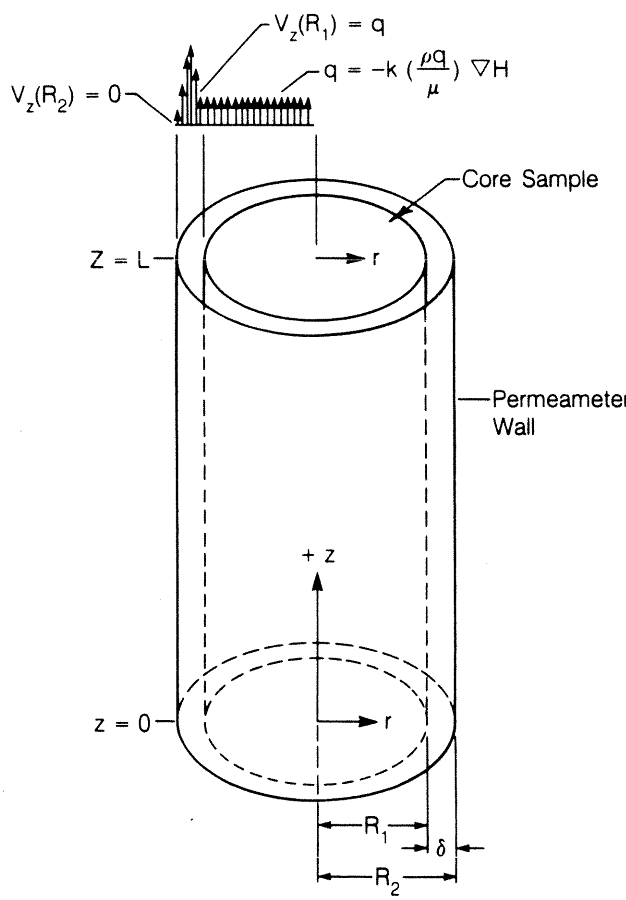

Fig. 1. Permeameter wall and soil core with an annular gap.

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where k is the permeability of the core. In the following developments, it will be more convenient to express core flow in terms of k rather than the saturated hydraulic conductivity, $K_s = k\rho g/\mu$, since use of the former parameter permits cancellation of fluid properties. The solution to Eq. [1] subject to Eq. [2a, b] is

$$v_z(r) = -\frac{\rho g R_2^2}{4\mu}\left[1 - \left(\frac{r}{R_2}\right)^2 - \left(\frac{1 - C^2 - (4k/R_2^2)}{\ln C}\right)\ln\frac{r}{R_2}\right]\frac{dH}{dz}. \quad [3]$$

The volumetric flow rate through the annulus, Q_{gap}, is obtained upon integrating Eq. [3] over the annular cross-section area,

$$Q_{\mathrm{gap}} = 2\pi\int_{R_1}^{R_2} r v_z(r)dr\,, \quad [4a]$$

resulting in

$$Q_{\mathrm{gap}} = -\frac{\rho g \pi R_2^4}{8\mu}\left[1 - C^4 + \frac{(1 - C^2)^2}{\ln C} - \frac{4k}{R_2^2}\left(2C^2 - \frac{(C^2 - 1)}{\ln C}\right)\right]\frac{dH}{dz}. \quad [4b]$$

In the limit of $k \rightarrow 0$, Eq. [3] and [4b] become equivalent to results presented in Bird et al. (1960), as expected.

The Flux through the Annulus Relative to that in the Core

The volumetric flow through a sample of permeability k is given by Darcy's law

$$Q_{\mathrm{core}} = -\pi R_1^2\frac{\rho g}{\mu}k\frac{dH}{dz}. \quad [5]$$

The quantity of interest is the ratio of flow through the annular space vs. the flow through the sample core. Dividing Eq. [4b] by Eq. [5] gives

$$\frac{Q_{\mathrm{gap}}}{Q_{\mathrm{core}}} = \frac{R_2^2}{8kC^2}\left[1 - C^4 + \frac{(1 - C^2)^2}{\ln C} - \frac{4k}{R_2^2}\left(\frac{1 - C^2}{\ln C} + 2C^2\right)\right]. \quad [6a]$$

The dependence of the gap to core flux ratio on the various system parameters is difficult to discern from Eq. [6a] due to its complexity. Even the apparently simple numerical evaluation of Eq. [6a] is susceptible to error due to both the behavior of $1 - C^n$ and $\ln C$ in the range of interest where $C \rightarrow 1$.

A clearer picture of the nature of the flux ratio can be obtained through the use of a small parameter ϵ defined by $\epsilon = (1 - C)$. From the definition of C and the gap width δ, ϵ is also equal to the ratio δ/R_2. Substituting ϵ into Eq. [6a] and expanding $\ln(1 - \epsilon)$ to at least four terms results in

$$\frac{Q_{\mathrm{gap}}}{Q_{\mathrm{core}}} \approx \frac{R_2^2}{k}\left[\frac{\frac{1}{6}\epsilon^4 + \frac{k}{R_2^2}\left(\epsilon^2 - \frac{1}{3}\epsilon^3 - \frac{1}{12}\epsilon^4\right)}{\epsilon - \frac{3}{2}\epsilon^2 + \frac{1}{3}\epsilon^3 + \frac{1}{12}\epsilon^4}\right], \quad [6b]$$

when terms higher than ϵ^4 are discarded. It is noted in passing that without retaining at least four terms in the expansion of $\ln(1 - \epsilon)$, all terms in the numerator of Eq. [6b] cancel. Since ϵ is a small parameter, Eq. [6b] can be simplified to

$$\frac{Q_{\mathrm{gap}}}{Q_{\mathrm{core}}} \approx \frac{R_2^2}{k}\left(\frac{1}{6}\epsilon^3 + \frac{k\epsilon}{R_2^2}\right), \quad [6c]$$

which in terms of the gap width δ is

$$\frac{Q_{\mathrm{gap}}}{Q_{\mathrm{core}}} \approx \frac{\delta^3}{6R_2k} + \frac{\delta}{R_2}. \quad [6d]$$

Comparison with a Parallel Plate Model

Due to the complexity of Eq. [4b], [6a], and [6b], it is of interest to consider a similar, though much simpler problem, for comparison with Eq. [6d]. The simpler model to be considered here is that of steady laminar flow between two parallel plates, with one plate surface velocity matched to a Darcy velocity. The system is depicted in Fig. 2. Flow occurs in response to a hydraulic head gradient along the z direction, in a gap of width δ, and transverse length W. Fluid at the surface defined by $x = 0$ is maintained at a constant velocity equated to a Darcy velocity. Fluid at the opposite wall at $x = \delta$ obeys the no-slip condition. The velocity profile in this case is

$$v_z(x) = -\frac{\rho g}{\mu}\left[-\frac{x^2}{2} + \left(\frac{\delta}{2} - \frac{k}{\delta}\right)x + k\right]\frac{dH}{dz}, \quad [7]$$

which when integrated over the cross-sectional area of the gap results in

$$Q_{\mathrm{gap}} = -W\left(\frac{\delta^3}{12} + \frac{k\delta}{2}\right)\frac{\rho g}{\mu}\frac{dH}{dz}. \quad [8]$$

A comparison between Eq. [6d] and [8] can be made by first equating the dimension W to $2\pi R_2$, the gap circumference in the annular flow problem. With this

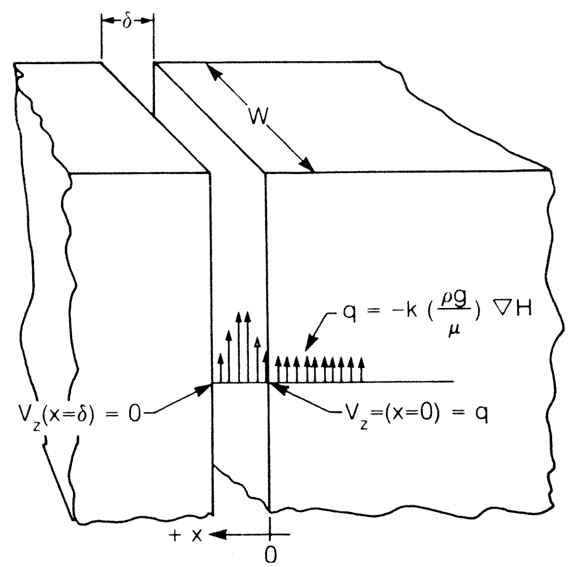

Fig. 2. Flow between parallel plates, with one plate consisting of a porous medium of finite permeability.

change, dividing Eq. [8] by Eq. [5] results in the parallel plate approximation to Eq. [6]. This result is

$$\frac{Q_{\text{gap}}}{Q_{\text{core}}} = \frac{2\pi R_2\left(\frac{\delta^3}{12} + \frac{k\delta}{2}\right)}{\pi R_1^2 k}, \qquad [9a]$$

which becomes

$$\frac{Q_{\text{gap}}}{Q_{\text{core}}} = \frac{\delta^3}{6R_2 k} + \frac{\delta}{R_2} \qquad [9b]$$

since $R_1 \approx R_2$. The agreement between Eq. [6d] and [9b] demonstrates that, when $\epsilon \rightarrow 0$, flow in an annular gap is well approximated by flow through a parallel plate gap.

Flow in Fractured Porous Media

The previous analysis of the parallel plate problem is related to the processes of flow in macropores between soil aggregates as well as to flow in fractured porous rock. To elaborate on this similarity, a simple model of a parallel planar gap bounded by a porous media of permeabilities k_1 and k_2 at $x = 0$ and $x = \delta$ respectively is considered in the following analysis. Fluid flow along the z direction is considered as before. In this case, the velocity profile is given by

$$v_z(x) = -\frac{\rho g}{\mu}\frac{dH}{dz}\left[-\frac{x^2}{2} - \left(-\frac{\delta}{2} + \frac{k_1 - k_2}{\delta}\right)x + k_1\right], \qquad [10]$$

which upon integration over the gap width δ and transverse width W gives

$$Q_{\text{gap}} = -\delta W\left[\frac{\delta^2}{12} + \frac{(k_1 + k_2)}{2}\right]\frac{\rho g}{\mu}\frac{dH}{dz}, \qquad [11a]$$

or when $k_1 = k_2 = k$,

$$Q_{\text{gap}} = -\delta W\left[\frac{\delta^2}{12} + k\right]\frac{\rho g}{\mu}\frac{dH}{dz}. \qquad [11b]$$

When either $k_1 \rightarrow 0$, or $k_2 \rightarrow 0$, Eq. [11a] becomes equivalent to Eq. [8] as expected. When $k \rightarrow 0$ at both surfaces, Eq. [11] results in the commonly used cubic flow relation with a parallel plate fracture permeability equal to $\delta^2/12$. This permeability is also commonly used to characterize flow in fractured porous media. However, from Eq. [11], it is evident that the true fracture permeability is of the form

$$k_{frac} = \left(\frac{\delta^2}{12}\right) + k \qquad [12]$$

where the second term accounts for finite velocities at the fracture surfaces. The relative error introduced by omission of this second term is equal to $12k/\delta^2$. This relative error is generally small due to preferential flow through the path of least resistance usually offered by fractures. It is this same phenomenon that gives rise to potentially large laboratory permeameter errors.

DISCUSSION

The inverse dependence of the ratio $Q_{\text{gap}}/Q_{\text{core}}$ in Eq. [6d] on the permeability k is reasonable. With higher permeability materials, errors due to annular flow become less significant. On the other hand, with very low permeability materials, annular flow becomes a very important source of experimental error. On the right-hand side of Eq. [6d], the result is dominated by the first term for reasonable values of the variables. This cubic dependence agrees with the cubic law flow result for flow within parallel plate gaps. The dependence of $Q_{\text{gap}}/Q_{\text{core}}$ on gap width is plotted in Fig. 3, for several values of k, with $R_2 = 36$ mm. The predicted flux ratios indicate that annular flow can contribute quite significantly to the overall flow within a permeameter, even at rather small gap widths. The curve for the moderately low permeability core with $k = 10^{-14}\text{m}^2$ indicates that an annular gap of only about 10 μm will lead to substantial errors in permeability measurements for such materials. Gap flow errors at even lower permeabilities can dominate measured flows since even micron-scale gaps will contribute significantly to the overall flow. It is emphasized that the assumptions used in the above calculations are very idealized. It is rather unlikely that an annular gap of uniform width will be found under most experimental conditions. Nevertheless, it appears likely that gaps along fractions of sample perimeters often occur. The calculations presented here serve to demonstrate the potential for large measurement errors, even with these likely fractional gaps.

With the currently increasing interest in measurement of k in low permeability soil and rock materials in association with hazardous waste disposal, annular flow in laboratory measurements becomes a potential source for very large errors. Annular flow errors in laboratory measurements on low permeability materials are likely for two reasons. First, due to the inverse k dependence of error effects demonstrated in Eq. [6d]

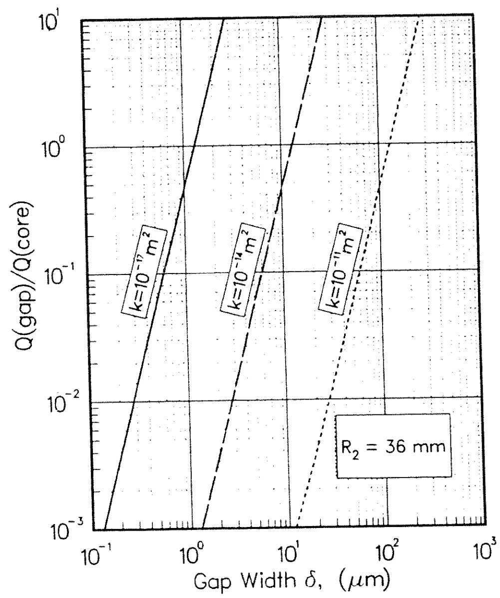

Fig. 3. The effect of gap width on the ratio of fluxes in the annulus to that in the core. Curves represent different permeabilities of the core sample. In this example the permeameter radius is 36 mm.

and [9], low k materials are inherently more susceptible to these errors. Second, in the case of rock samples that lack the plasticity found in unconsolidated soils, core walls are much less capable of conforming to the surfaces of a permeameter.

Observations of preferential water and solute flow in field soils (e.g., Beven and Germann, 1982; Richter and Jury, 1986) appear to arise from the same phenomenon considered here. Saturated water flow through annular permeameter gaps, through fractures in porous rock, and through macropores in field soils generally results in high flux densities and short solute travel times. Another related process is that of water and solute collection in lysimeters. Interpretation of field lysimeter data can be complicated due to the possibility of wall flow effects overwhelming macropore flow (Richter and Jury, 1986). The underlying effect in all of the abovementioned processes is due ultimately to the parabolic nature of fluid velocity profiles at the pore (or fracture) scale.

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